Borel ideals and topological Ramsey spaces

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Summary

Introduction

- Topological Ramsey space theory is concerned with infinite dimensional colorings.
- The prototypical example of a TRS is the Ellentuck space.
- Ellentuck space with the almost inclusion order is forcing equivalent to $\mathcal{P}(\omega)/Fin$ and forces a Ramsey ultrafilter.
- Todorcevic proved using large cardinals that Ramsey ultrafilters are generic over $L(\mathbb{R})$ for the poset $\mathcal{P}(\omega)/Fin$.
- Baumgartner and Laver showed that Ramsey ultrafilters are preserved by both side-by-side and iterated Sacks forcing.

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Ultrafilters and Topological Ramsey spaces

- (Di Prisco, Mijares, Nieto) If there exists a super compact cardinal, then every selective coideal $\mathcal{U} \subseteq \mathcal{R}$ is $(\mathcal{R}, \leq^*)$-generic over $L(\mathbb{R})$.
- Zheng states some condition on TRS to guarantee that the generic filter is Sacks indestructible.
- (Navarro) Every topological Ramsey space forces a Ramsey ultrafilter.
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Motivation

- Several ultrafilters forced by σ-closed partial orders are also forced by topological Ramsey spaces.
- Dobrinen and Todorcevic use topological Ramsey Spaces to find the precise structure of the Tukey order.
- Dobrinen and Navarro use topological Ramsey spaces to calculate exact Ramsey degrees of several ultrafilters.
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Ideals

- Recall that an ideal on a set X is a collection $\mathcal{I} \subset \mathcal{P}(X)$ which is closed under subsets and finite unions.
- We are interested in ideals on countable sets so we think of ideals as subsets of ω.
- We can identify $\mathcal{P}(\omega)$ with 2^ω so we think of ideals as subsets of 2^ω with the inherited topology.
- $\mathcal{I}^+$ is the collection of subsets of ω that don't belong to $\mathcal{I}$. Members of $\mathcal{I}^+$ are called $\mathcal{I}$-positive sets.

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Ideals

Definition

An ideal $\mathcal{I}$ is P^+ if for every decreasing sequence $\{X_n : n \in \omega\}$ of $\mathcal{I}$-positive sets there is some $X \in \mathcal{I}^+$ such that $X \subseteq^* X_n$ for every $n \in \omega$.

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TRS ideals

We are interested in ideals that are related to TRS in the same sense that Fin is related to the Ellentuck space.

Definition

Let $\mathcal{I}$ be an ideal on ω, we say that $\mathcal{I}$ is a *Topological Ramsey Space* ideal if there is some topological Ramsey space $\mathcal{R} \subseteq \mathcal{I}^+$ such that for every $X \in \mathcal{I}^+$ there is some $Y \in \mathcal{R}$ such that $Y \subseteq X$.

Examples

- $\mathcal{I}$ =Fin is a Topological Ramsey Space ideal.
- The *eventually different ideal* $\mathcal{ED}$ consists of sets $A \subseteq \omega \times \omega$ such that

$$(\exists m, n \in \omega)(\forall k > m)(|\{l \in \omega : (k, l) \in A\}| < \omega).$$

  $\mathcal{ED}$ is a TRS ideal.
- The ideal $Fin \times Fin$ contains sets $A \subseteq \omega \times \omega$ such that

$$|\{m \in \omega : |\{n \in \omega : (m, n) \in A\}| = \omega\}| < \omega.$$

  $Fin \times Fin$ is a TRS ideal.
- The ideal of convergent sequences ideal on $\mathbb{Q} \cap (0, 1)$ is a TRS ideal.
- Summable ideal $\mathcal{I}_{\frac{1}{n}}$ is not a TRS ideal.
- Many other ideals are TRS ideals.

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Definition

Let $\mathcal{I}$ be a Topological Ramsey Space ideal with $\mathcal{R}$ a topological Ramsey space associated to $\mathcal{I}$ and $\mathcal{AR}$ the collection of finite approximations of members of $\mathcal{R}$. We say that

- $\mathcal{I}$ has the *Independent Sequence Property* if for every $s \in \mathcal{AR}$ and $X \in \mathcal{R}$ there is some $t \in \mathcal{R}$ such that $s \sqsubset t$ and $t \setminus s \subset X$.
- $\mathcal{I}$ has the *Dependence Property* if for every $s \in \mathcal{AR}$ and $X \in \mathcal{R}$ such that $s \not\subseteq X$ there are no $t \in \mathcal{R}$ such that $s \sqsubset t$ and $t \setminus s \subset X$.

We say that $\mathcal{I}$ is *weakly homogeneous* if $\mathcal{I}$ has the Independent Sequence Property or the Dependence Property.

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HL ideals

If p is a Sacks tree and $A \subseteq \omega$, we denote by $p \restriction A$ the collection of nodes $s \in p$ such that $s \in 2^n$ for some $n \in A$.

Definition

We say an ideal $\mathcal{I}$ is Halpern Läuchli (HL) if for every coloring $c : 2^{<\omega} \to 2$ there are $p \in \mathbb{S}acks$ and $A \in \mathcal{I}^+$ such that $p \restriction A$ is homogeneous.

Proposition (Hrusak, Navarro)

An ideal $\mathcal{I}$ is HL if and only if the $\mathcal{P}(\omega)/\mathcal{I}$-generic filter is Sacks indestructible.

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Proposition

If $\mathcal{I}$ is a P^+-ideal then $\mathcal{I}$ is HL.

Proof. If $\mathcal{I}$ is a P^+-ideal and $\mathcal{U}$ is a $\mathcal{P}(\omega)/\mathcal{I}$-generic ultrafilter then $\mathcal{U}$ is a P-point. Since P-points are Sacks indestructible, by the previous Proposition the conclusion follows.

Corollary

If $\mathcal{I}$ is a F_σ ideal then $\mathcal{I}$ is HL.

Proof. This is because F_σ ideals are P^+.

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Let $\mathcal{I}$, $\mathcal{J}$ be ideals on ω. We way that $\mathcal{I}$ is Katetov below $\mathcal{J}$ ($\mathcal{I} \leq_K \mathcal{J}$) if there is some map $f : \omega \to \omega$ such that for every $X \in \mathcal{I}$, $f^{-1}[X] \in \mathcal{J}$.

Members of the ideal $\mathcal{ED}_{fin}$ are members of $\mathcal{ED}$ which are contained below the diagonal.

Proposition (Chodounsky, Guzman, Hrusak)

If $\mathcal{I}$ is not HL then there exists some $X \in \mathcal{I}^+$ such that $\mathcal{ED}_{fin} \leq_K \mathcal{I} \restriction X$.

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If $\mathcal{I}$ is an ideal with the Independent Sequence Property then $\mathcal{I}$ is P^+.

Work in progress

If $\mathcal{I}$ is an ideal with the Dependence Property then there is no $X \in \mathcal{I}^+$ such that $\mathcal{ED}_{fin} \leq_K \mathcal{I} \upharpoonright X$.

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Conjecture

If $\mathcal{I}$ is a weakly homogeneous TRS ideal and $\mathcal{U}$ is a $\mathcal{P}(\omega)/\mathcal{I}$-generic ultrafilter then $\mathcal{U}$ is Sacks indestructible.

Applications

The *asymptotic density zero* ideal $\mathcal{Z}$ is the ideal

$$\mathcal{Z} = \{A \subseteq \omega : lim_{n\to\infty} \frac{|A \cap n|}{n} = 0\}.$$

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$\mathcal{Z}$ is not HL.

If the conjecture is true then the ideal $\mathcal{Z}$ is not a TRS ideal.

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Theorem (Zheng)

Ultrafilters forced by Gowers space FIN, topological Ramsey spaces $\mathcal{R}_\alpha$ and finite dimensional Ellentuck spaces $\mathcal{E}_k$ are Sacks indestructible.

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